Calculation of Arithmetic Mean – Individual Observations

The arithmetic mean (A.M.) of a set of n observations $X_1, X_2, \ldots, X_n$ (not necessarily all distinct), denoted by $\bar{X}$, is given by

$$\bar{X} = \frac{X_1 + X_2 + \ldots + X_n}{n} = \frac{\sum X}{n}$$

The summation notation $\sum X$ is an abbreviated form of the more general notation $\sum_{i=1}^{n} X_i$.

We will use the abbreviated form $\sum X$ to indicate the sum of all the numbers being considered.

EXAMPLE 1. *The following figures give the marks of 10 students in a class test:*

Marks obtained : 12 8 17 13 15 9 18 11 6 1

Find the arithmetic mean.

SOLUTION. The arithmetic mean of the marks is determined by finding the sum of all the marks and then dividing this total by 10. Thus

$$\bar{X} = \frac{\sum X}{n} = \frac{12+8+17+13+15+9+18+11+6+1}{10} = \frac{110}{10} = 11$$

Short-cut Method. It may be pointed out that if the values of X are very large, the computation of arithmetic mean can be done by using what is known as *short-cut method.* The various steps involved in the computation of arithmetic mean by short-cut method are as follows:

Step 1. *Choose an arbitrary number A, called an assumed mean. Any number can be chosen as an assumed mean. However, it is usually taken as the value of X which corresponds to the middle part of the distribution. Moreover, A need not necessarily be one of the values of X.*

Step 2. *Compute $d = X - A$, deviation of X from A. Algebraic signs + or – are to be taken with the deviations.*

Step 3. *The arithmetic mean is given by*

$$\bar{X} = A + \frac{\sum d}{n}.$$

EXAMPLE 2. *The following figures show the heights in cms of 7 students chosen at random:*

164, 159, 167, 169, 165, 170, 168.

Calculate the arithmetic mean of heights by (a) Direct method (b) Short-cut method.

SOLUTION. **CALCULATION OF ARITHMETIC MEAN**

S.No.	Height (in cm) X	$A = 165$ $d = X - A$
1	164	-1
2	159	-6
3	167	2
4	169	4
5	165	0
6	170	5
7	168	3
$n = 7$	$\sum X = 1162$	$\sum d = 7$

(*a*) *Direct Method* : $\bar{X} = \frac{\sum X}{n} = \frac{1162}{7} = 166$ cm.

(*b*) *Short-cut Method* : $\boxed{\bar{X} = A + \frac{\sum d}{n}} = 165 + \frac{7}{7} = 165 + 1 = 166$ cm.

Calculation of Arithmetic Mean - Discrete Series

In case of discrete series where the variable X takes the values $X_1, X_2, \ldots, X_n$ with respective frequencies $f_1, f_2, \ldots, f_n$, the arithmetic mean can be calculated by applying

(*i*) *Direct Method*, or

(*ii*) *Short-cut Method.*

Direct Method. According to this method, the *A.M.* is given by

$$\bar{X} = \frac{f_1X_1 + f_2X_2 + \ldots + f_nX_n}{f_1 + f_2 + \ldots + f_n} = \frac{\sum fX}{\sum f} = \frac{\sum fX}{N}$$

where $N = \sum f$ = total frequency.

Short-cut Method. According to this method, arithmetic mean is given by

$$\bar{X} = A + \frac{\sum fd}{N}$$

where A = assumed mean, $d = X - A$ and $N = \sum f$.

EXAMPLE 3. *Calculate the arithmetic mean for the following discrete frequency distribution:*

X	:	20	30	40	50	60	70
f	:	8	12	20	10	6	4

SOLUTION. **CALCULATION OF ARITHMETIC MEAN**

X	*f*	*fX*
20	8	160
30	12	360
40	20	800
50	10	500
60	6	360
70	4	280
	$N = \sum f = 60$	$\sum fX = 2460$

$$\bar{X} = \frac{\sum fX}{\sum f} = \frac{2460}{60} = 41$$

EXAMPLE 4. *The following data give the daily earnings (in Rs.) of 20 workers in a factory:*

Daily earnings (in Rs.)	:	100	140	170	200	250
No. of workers	:	5	2	6	4	3

Calculate the average daily earnings using: (a) Direct Method (b) Short-cut Method.

SOLUTION. **CALCULATION OF ARITHMETIC MEAN**

Daily earnings *X*	*No. of workers* *f*	*fX*	*A* = 170 *d* = *X* − *A*	*fd*
100	5	500	−70	−350
140	2	280	−30	−60
170	6	1020	0	0
200	4	800	30	120
250	3	750	80	240
	$N = \sum f = 20$	$\sum fX = 3350$		$\sum fd = -50$

(*a*) ***Direct Method*** **:** According to this method, the average daily earnings is:

$$\bar{X} = \frac{\sum fX}{\sum f} = \frac{3350}{20} = \text{Rs. } 167.50$$

(*b*) ***Short-cut Method*** **:** According to this method, the average daily earnings is:

$$\bar{X} = A + \frac{\sum fd}{N} = 170 + \frac{-50}{20} = 170 - 2.5 = \text{Rs. } 167.50$$

Calculation of Arithmetic Mean-Continuous Series

In case of continuous series, the arithmetic mean may be computed by applying any of the following methods:

(i) *Direct Method,*

(ii) *Short-cut Method,*

(iii) *Step-deviation Method.*

Direct Method. If $X_1, X_2, \ldots, X_n$ are the class marks (or mid-values) of a set of grouped data with corresponding class frequencies $f_1, f_2, \ldots f_n$, then according to the direct method, arithmetic mean is given by

$$\bar{X} = \frac{f_1X_1 + f_2X_2 + \ldots + f_nX_n}{f_1 + f_2 + \ldots + f_n} = \frac{\sum fX}{\sum f} = \frac{\sum fX}{N}$$

where $N = \sum f$ is the total frequency.

Short-cut Method. According to this method, arithmetic mean is given by

$$\bar{X} = A + \frac{\sum fd}{N}$$

where A = assumed mean, $d = X - A$, deviation of mid-value from assumed mean, and $N = \sum f$ = total frequency.

Step-deviation Method. In case of grouped or continuous frequency distribution with class intervals of equal size, the calculation of arithmetic mean can further be simplified by taking

$$u = \frac{X - A}{h}$$

where X is the mid-value and h is the common size (or width) of the class intervals. According to this method, the arithmetic mean is given by

$$\bar{X} = A + \frac{\sum fu}{N} \times h$$

EXAMPLE 5. *Compute the arithmetic mean from the following frequency distribution:*

Marks	:	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	:	5	7	8	14	10	6

SOLUTION. **CALCULATION OF ARITHMETIC MEAN**

Marks	*Mid-value* X	*No. of students* f	fX
0 – 10	5	5	25
10 – 20	15	7	105
20 – 30	25	8	200
30 – 40	35	14	490
40 – 50	45	10	450
50 – 60	55	6	330
		$N = \sum f = 50$	$\sum fX = 1600$

$\therefore$ Arithmetic Mean is : $\bar{X} = \frac{\sum fX}{\sum f} = \frac{1625}{50} = 32.5$

EXAMPLE 6. *Calculate the arithmetic mean from the following frequency distribution:*

Marks :	*0 – 10*	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*
No. of students :	*10*	*9*	*25*	*30*	*16*	*10*

SOLUTION. **CALCULATION OF ARITHMETIC MEAN**

Marks	*Mid-value* X	*No. of students* f	fX	$A=35$ $d=X-A$	$h=10$ $u=\frac{X-A}{h}$	fd	fu
0 – 10	5	10	50	– 30	– 3	– 300	– 30
10 – 20	15	9	135	– 20	– 2	– 180	– 18
20 – 30	25	25	625	– 10	– 1	– 250	– 25
30 – 40	35	30	1050	0	0	0	0
40 – 50	45	16	720	10	1	160	16
50 – 60	55	10	550	20	2	200	20
		$N=\sum f$ = 100	$\sum fX$ = 3130			$\sum fd$ = – 370	$\sum fu$ = – 37

Direct Method : $\bar{X} = \frac{\sum fX}{N} = \frac{3130}{100} = 31.30$

Short-cut Method : $\bar{X} = A + \frac{\sum fd}{N} = 35 + \frac{-370}{100} = 35 - 3.70 = 31.30$

Step-deviation Method : $\bar{X} = A + \frac{\sum fu}{N} \times h = 35 + \frac{-37}{100} \times 10$

$= 35 - 3.70 = 31.30.$

EXAMPLE 7. *Calculate mean from the following data :*

Marks	*No. of students*	*Marks*	*No. of students*
Less than 10	*4*	*Less than 50*	*96*
Less than 20	*16*	*Less than 60*	*112*
Less than 30	*40*	*Less than 70*	*120*
Less than 40	*76*	*Less than 80*	*125*

SOLUTION. We are given 'less than' cumulative frequency distribution. We shall first convert it into an ordinary frequency distribution and then calculate mean.

CALCULATION OF ARITHMETIC MEAN

Marks	Mid-value X	No. of students f	$u=\frac{X-45}{10}$ (A = 45, h = 10)	fu
0 – 10	5	4	–4	–16
10 – 20	15	12	–3	–36
20 – 30	25	24	–2	–48
30 – 40	35	36	–1	–36
40 – 50	45	20	0	0
50 – 60	55	16	1	16
60 – 70	65	8	2	16
70 – 80	75	5	3	15
			$N=\sum f=125$	$\sum fu=-105$

$$\bar{X}=A+\frac{\sum fu}{N}\times h = 45+\frac{-105}{125}\times 10 = 45-8.4 = 36.60$$

EXAMPLE 8. *The following table gives the life-time in hours of 400 radio tubes of a certain make.*

Life-time (in hours)	No. of tubes	Life-time (in hours)	No. of tubes
Less than 300	0	Less than 800	265
Less than 400	20	Less than 900	324
Less than 500	60	Less than 1000	374
Less than 600	116	Less than 1100	392
Less than 700	194	Less than 1200	400

Calculate the mean life-time of radio tubes. Imp.

SOLUTION. The data is given in the form of a cumulative frequency distribution. To calculate the mean, we shall first convert it into an ordinary frequency distribution as shown below:

CALCULATION OF ARITHMETIC MEAN

Class Interval	Frequency (f)	Mid-value (X)	$u=\frac{X-750}{100}$ (A = 750, h = 100)	fu
300 – 400	20 – 0 = 20	350	– 4	– 80
400 – 500	60 – 20 = 40	450	– 3	– 120
500 – 600	116 – 60 = 56	550	– 2	– 112
600 – 700	194 – 116 = 78	650	– 1	– 78
700 – 800	265 – 194 = 71	750	0	0
800 – 900	324 – 265 = 59	850	1	59
900 – 1000	374 – 324 = 50	950	2	100
1000 – 1100	392 – 374 = 18	1050	3	54
1100 – 1200	400 – 392 = 8	1150	4	32
	$N=\sum f=400$			$\sum fu=-145$

$$\bar{X}=A+\frac{\sum fu}{N}\times h = 750+\left(\frac{-145}{400}\right)\times 100 = 750\times(-36.25)$$
$$= 713.5 \text{ hours.}$$

REMARK. It may be remarked that even if the data is given in the form of a grouped frequency distribution with '*inclusive type*' classes, it is not necessary to adjust the classes for calculating arithmetic mean because the mid-values remain the same whether or not the adjustment is made.

EXAMPLE 10. *Given below is the distribution of marks obtained by 140 students in an examination:*

Marks :	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
No. of students :	7	15	18	25	30	20	16	7	2

Find the mean of the distribution.

SOLUTION.

CALCULATION OF ARITHMETIC MEAN

Marks	*Mid-value* X	*No. of students* f	$u=\frac{X-54.5}{10}$	fu
10 – 19	14.5	7	– 4	– 28
20 – 29	24.5	15	– 3	– 45
30 – 39	34.5	18	– 2	– 36
40 – 49	44.5	25	– 1	– 25
50 – 59	54.5	30	0	0
60 – 69	64.5	20	1	20
70 – 79	74.5	16	2	32
80 – 89	84.5	7	3	21
90 – 99	94.5	2	4	8
		$N=\sum f = 140$		$\sum fu = -53$

Mean of the distribution is:

$$\bar{X} = A+\frac{\sum fu}{N}\times h = 54.5+\frac{-53}{140}\times 10 = 54.5-3.79 = 50.71.$$

The mean of the following frequency distribution is 50. But the frequencies f_1.

EXAMPLE 11. *The mean of the following frequency distribution is 50. But the frequencies f_1 and f_2 in classes 20 – 40 and 60 – 80 are missing. Find the missing frequencies.*

Class :	*0 – 20*	*20 – 40*	*40 – 60*	*60 – 80*	*80 – 100*	*Total*
Frequency :	*17*	f_1	*32*	f_2	*19*	*120*

[Delhi Univ. B. Com., 1999]

SOLUTION. **CALCULATION OF MISSING FREQUENCIES**

Class	*Mid-value* X	*Frequency* f	$u=\frac{X-50}{20}$ $(A=50, h=20)$	fu
0 – 20	10	17	–2	–34
20 – 40	30	f_1	–1	$-f_1$
40 – 60	50	32	0	0
60 – 80	70	f_2	1	f_2
80 – 100	90	19	2	38
		$N=\sum f = 68+f_1+f_2$		$\sum fu = 4-f_1+f_2$

We are given $N = 120 \Rightarrow 68+f_1+f_2 = 120 \Rightarrow f_1+f_2 = 52$ ... (1)

Using the step-deviation method for calculating mean, we obtain

$$\bar{X} = A+\frac{\sum fu}{N}\times h$$

i.e., $$50 = 50 + \frac{4 - f_1 + f_2}{120} \times 20$$

$$\Rightarrow \quad \frac{4 - f_1 + f_2}{120} = 0 \Rightarrow 4 - f_1 + f_2 = 0 \Rightarrow f_1 - f_2 = 4 \quad \text{... (2)}$$

Adding (1) and (2), we get $2f_1 = 56 \Rightarrow f_1 = 28.$

Substituting $f_1 = 28$ in (1), we get $f_2 = 24$.

EXAMPLE 12. *For the following data find the missing frequency if the arithmetic mean is 33.*

Marks :	*0 – 10*	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*
No. of students :	*10*	*15*	*30*	–	*25*	*20*

[C.A. Foundation, Nov. 2000]

SOLUTION. Let the missing frequency be x.

Marks	*Mid-value* X	*No. of Students* f	$u = \frac{X - 35}{10}$ $(A = 35, h = 10)$	fu
0 – 10	5	10	–3	–30
10 – 20	15	15	–2	–30
20 – 30	25	30	–1	–30
30 – 40	35	x	0	0
40 – 50	45	25	1	25
50 – 60	55	20	2	40
		$N = \sum f = 100 + x$		$\sum fu = -25$

Using the step-deviation method $\bar{X} = A + \frac{\sum fu}{N} \times h$

for calculating the mean, we obtain $33 = 35 + \frac{-25}{100 + x} \times 10 \Rightarrow \frac{250}{100 + x} = 35 - 33 = 2$

$$\Rightarrow \quad 200 + 2x = 250 \quad \Rightarrow \quad 2x = 50 \quad \Rightarrow \quad x = 25$$

Thus the missing frequency is 25.

Correcting Incorrect Mean

The last property is quite useful to find corrected mean whenever one, two or more of the observations were wrongly copied down. For example, suppose we have computed the mean $\bar{X}$ of n observations and later on it is found that two observations, say, X_1 and X_2, were wrongly copied down as X'_1 and X'_2. It is now required to compute the corrected mean by replacing the wrong observations by the correct ones. By using Property 5, we can first obtain the uncorrected sum of the observations which is given by $n\bar{X}$. From this, we subtract the wrong observations X'_1 and X'_2 and add the corresponding correct observations X_1 and X_2 to get the corrected sum

$$n\bar{X} - (X'_1 + X'_2) + (X_1 + X_2)$$

Dividing this by n, we get the corrected mean.

In general, if r observations are misread as $X'_1, X'_2, \ldots, X'_r$ while correct observations are $X_1, X_2, \ldots . X_r$, then the corrected sum of observations is given by

$$n\bar{X} - (X'_1 + X'_2 + \ldots + X'_r) + (X_1 + X_2 + \ldots + X_r)$$

EXAMPLE 25. *The mean marks of 100 students were found to be 40. Later on it was discovered that, a score of 53 was misread as 83. Find the correct mean corresponding to the correct score.*

SOLUTION. We are given $n = 100$ and $\bar{X} = 40$

$$\because \quad \bar{X} = \frac{\sum X}{n} \quad \therefore \quad \sum X = n\bar{X} = 100 \times 40 = 4000$$

But this is not the correct $\sum X$. In fact,

$$\text{Correct } \sum X = 4000 - \text{wrong Score} + \text{correct Score}$$

$$= 4000 - 83 + 53 = 3970$$

$$\therefore \quad \text{Correct } \bar{X} = \frac{\text{Corrected } \sum X}{n} = \frac{3970}{100} = 39.7$$

Thus, the correct mean is 39.7.

3.6 COMBINED MEAN

If the arithmetic means of two or more sets of data are known, then we can also obtain the arithmetic mean of the combined data. For example, if n_1, and n_2 are the number of observations and $\bar{X}_1, \bar{X}_2$ are the respective means of two sets of data, then the mean, $\bar{X}$ or $\bar{X}_{12}$, of the combined data with $n_1 + n_2$ observations is given by

$$\bar{X} = \bar{X}_{12} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

The result can be generalised to more than two sets of data. For example, if $\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_k$ be the mean of different sets of data and $n_1, n_2, \ldots, n_k$ be the number of observations in each set, then the arithmetic mean of the combined data with $n_1 + n_2 + \ldots + n_k$ observations is given by

$$\bar{X} = \bar{X}_{1,2,\ldots,k} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2 + \ldots + n_k\bar{X}_k}{n_1 + n_2 + \ldots + n_k}$$

We shall now illustrate the application of the above formula with the help of following examples

EXAMPLE 27. *Find out the combined mean from the following data :*

	Series X	*Series Y*
Arithmetic Mean	*12*	*20*
Number of Items	*80*	*60*

[Delhi Univ. B. Com. 1978]

SOLUTION. We are given : $\bar{X}_1 = 12$ $\quad \bar{X}_2 = 20$

$n_1 = 80$ $\quad n_2 = 60$

$$\therefore \quad \text{Combined Mean} \quad (\bar{X}) = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

$$= \frac{80 \times 12 + 60 \times 20}{80 + 60} = \frac{960 + 1200}{140} = \frac{2160}{140} = 15.43.$$

EXAMPLE 28. *B. Com. (P) IIIrd year has three Sections A, B and C with 50, 40 and 60 students respectively. The mean marks for the three sections were determined as 85, 60 and 65 respectively. However, marks of a student of Section A were wrongly recorded as 50 instead*

[Delhi Univ. B.Com. 1995]

SOLUTION. The information in this problem may be expressed as follows :

	A	B	C
Number of students	$n_1 = 50$	$n_2 = 40$	$n_3 = 60$
Arithmetic Mean	$\bar{X}_1 = 85$	$\bar{X}_2 = 60$	$\bar{X}_3 = 65$
$\therefore$	$\sum X_1 = 4250$	$\sum X_2 = 2400$	$\sum X_3 = 3900$

$\because$ Marks of a student of Section A were wrongly recorded as 50 instead of 0, therefore $\sum X_1$ needs to be corrected. In fact,

$$\text{Corrected } \sum X_1 = 4250 - 50 + 0 = 4200$$

$$\therefore \quad \text{Corrected } \bar{X}_1 = \frac{\text{Corrected } \sum X_1}{n_1} = \frac{4200}{50} = 84$$

$\therefore$ Mean marks of all the three sections put together :

$$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2 + n_3\bar{X}_3}{n_1 + n_2 + n_3} = \frac{50 \times 84 + 40 \times 60 + 60 \times 65}{50 + 40 + 60}$$

$$= \frac{4200 + 2400 + 3900}{150} = \frac{10500}{150} = 70.$$

EXAMPLE 29. *A distribution consists of three components with total frequencies of 200, 250 and 300 having means 25, 10 and 15 respectively. Find the mean of the combined distribution.* **[Delhi Univ. B. Com. 2006]**

SOLUTION. We are given : $n_1 = 200 \quad n_2 = 250 \quad n_3 = 300$

$$\bar{X}_1 = 25 \quad \bar{X}_2 = 10 \quad \bar{X}_3 = 15$$

Mean of the combined distribution is given by

$$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2 + n_3\bar{X}_3}{n_1 + n_2 + n_3} = \frac{200 \times 25 + 250 \times 10 + 300 \times 15}{200 + 250 + 300}$$

$$= \frac{5000 + 2500 + 4500}{750} = \frac{12000}{750} = 16.$$

EXAMPLE 30. *Fifty student took up a test. The result of those who passed the test is given below:*

Marks :	4	5	6	7	8	9
Number of Students :	8	10	9	6	4	3

In the average for all 50 students was 5.16 marks, find the average of those who failed.

[Delhi Univ. B.Com. 2001]

SOLUTION. Let the average marks of students who passed the test be denoted by $\bar{X}_1$, and average marks of those who failed be denoted by $\bar{X}_2$. Then $\bar{X}_1$ is calculated as follows :

CALCULATION FOR $\bar{X}_1$

Marks (X_1)	No. of students (f)	fX_1
4	8	32
5	10	50
6	9	54
7	6	42
8	4	32
9	3	27
	$N = \sum f = 40$	$\sum fX_1 = 237$

$$\therefore \quad \bar{X}_1 = \frac{\sum fX_1}{N} = \frac{237}{40} = 5.925$$

If average marks of all students be denoted by $\bar{X}$, then we are given $\bar{X} = 5.16$. Thus, in terms of usual notation, we are given the following information :

	Passed	*Failed*	*Total*
Number of Students	$n_1 = 40$	$n_2 = 10$	$n_1 + n_2 = 50$
Mean Marks	$\bar{X}_1 = 5.925$	$\bar{X}_2 = ?$	$\bar{X}_3 = 5.16$

Applying the formula $\bar{X} = \dfrac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$ for the combined mean, we obtain

$$5.16 = \frac{40 \times 5.925 + 10\bar{X}_2}{50} \quad \Rightarrow \quad 258 = 237 + 10\bar{X}_2$$

$$\Rightarrow \quad 10\,\bar{X}_2 = 258 - 237 = 21 \quad \text{or} \quad \bar{X}_2 = 2.1$$

$\therefore$ Average marks of the students who failed $= 2.1$

ALITER : Total marks of all the 50 students $= 50 \times 5.16 = 258$

Total marks of 40 students who passed the test $= \sum fX_1 = 237$

$\therefore$ Total marks of the remaining 10 students who failed $= 258 - 237 = 21$

$\therefore$ Average marks of those who failed $= \dfrac{21}{10} = 2.1.$

EXAMPLE 31. *The mean wage of 100 workers in a factory running in two shifts of 60 and 40 workers respectively is Rs. 38. The mean wage of 60 workers working in the morning shift is Rs. 40. Find the mean wage of the workers working in the evening shift.*

[Delhi

SOLUTION. We are given :

	Morning Shift	*Evening Shift*	*Combined*
Number of Workers	$n_1 = 60$	$n_2 = 40$	$n_1 + n_2 = 100$
Mean	$\bar{X}_1 = 40$	$\bar{X}_2 = ?$	$\bar{X} = 38$

Applying the formula $\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$ for the combined mean, we get

$$38 = \frac{60 \times 40 + 40 \times \bar{X}_2}{100}$$

$$\Rightarrow \quad 3800 = 2400 + 40\,\bar{X}_2 \Rightarrow 40\,\bar{X}_2 = 1400 \Rightarrow \bar{X}_2 = 35$$

$\therefore$ The mean wage of 40 workers working in the evening shift is Rs. 35.

EXAMPLE 32. *The mean weight of 150 students (boys and girls) in a class is 60 kg. The mean weight of boy students is 70 kg and that of girl students is 55 kg. Find the number of boys and girls in the class.* **[C.A. PEE-I, 2002]**

SOLUTION. Let n_1 be the number of boys and n_2 be the number of girls in the class. Then we are given

	Boys	*Girls*	*Combined*
Number of Students	n_1	n_2	$n_1 + n_2 = 150$
Mean Weight	$\bar{X}_1 = 70$	$\bar{X}_2 = 55$	$\bar{X} = 60$

Applying the formula $\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$ for the combined mean, we get

$$60 = \frac{70n_1 + 55n_2}{150}$$

$$\Rightarrow \quad 70n_1 + 55n_2 = 60 \times 150 = 9000 \quad \Rightarrow \quad 14n_1 + 11n_2 = 1800$$

Solving the system of equations :

$$n_1 + n_2 = 150 \quad \text{and} \quad 14n_1 + 11n_2 = 1800$$

for n_1 and n_2, we obtain $n_1 = 50$ and $n_2 = 100$

$\therefore$ Number of boy students = 50, and number of girl students = 100

EXAMPLE 33. *In a certain examination, the average grade of all the students in Section A is 68.4 and the average grade of those in Section B is 71.2. The average grade of all the students in Sections A and B combined is 70. Find the ratio of the number of students in Section A to the number of students in Section B.*

SOLUTION. Let n_1 and n_2 denote the number of students in Section A and Section B respectively. Then in terms of usual notations, we are given :

	Section A	*Section B*	*Combined*
Number of Students	n_1	n_2	$n_1 + n_2$
Average grade	$\bar{X}_1 = 68.4$	$\bar{X}_2 = 71.2$	$\bar{X} = 70$

Applying the formula $\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$ for the combined mean, we get

$$70 = \frac{68.4n_1 + 71.2n_2}{n_1 + n_2}$$

$$\Rightarrow \quad 70n_1 + 70n_2 = 68.4\,n_1 + 71.2\,n_2$$

$$\Rightarrow \quad 1.6n_1 = 1.2n_2 \quad \Rightarrow \quad \frac{n_1}{n_2} = \frac{1.2}{1.6} = \frac{3}{4}$$

Thus the number of students in Section *A* and Section *B* are in the ratio $3:4$.

EXAMPLE 34. *The mean annual salary of all employees in a company is Rs. 25,000. The mean salary of male and female employees is Rs. 27,000 and Rs. 17,000 respectively. Find the percentage of males and females employed by the company.*

[C.A. Foundation, Nov. 1995]

SOLUTION. Let n_1 and n_2 denote, respectively, the number of males and females employed by the company. Then in terms of usual notations, we are given :

	Males	*Females*	*Combined*
Number of Employees	n_1	n_2	$n_1 + n_2$
Mean Salary	$\bar{X}_1 = 27000$	$\bar{X}_2 = 17000$	$\bar{X} = 25000$

Applying the formula $\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$ for the combined mean, we get

$$25{,}000 = \frac{27000n_1 + 17000n_2}{n_1 + n_2}$$

$$\Rightarrow \quad 25{,}000n_1 + 25{,}000n_2 = 27{,}000n_1 + 17{,}000n_2$$

$$\Rightarrow \quad 2000n_1 = 8000n_2 \quad \Rightarrow \quad \frac{n_1}{n_2} = \frac{4}{1}$$

$\Rightarrow$ Number of male and female employees are in the ratio 4 : 1.

$$\therefore \quad \%\text{ of male employees} = \frac{4}{4+1} \times 100 = 80\%$$

and $$\%\text{ of female employees} = \frac{1}{4+1} \times 100 = 20\%$$

EXAMPLE 35. *100 students appeared for an examination. The results of those who failed are given below :*

Marks :	5	10	15	20	25	30	*Total*
Number of Students :	4	6	8	7	3	2	30

If the average marks of all students were 68.6, find out average marks of those who passed.

[Delhi Univ. B.Com. (H) 2008]

SOLUTION. Let the average marks of students who passed the examination be denoted by

$\bar{X}_1$ and average marks of those who failed be denoted by $\bar{X}_2$. Then $\bar{X}_2$ is calculated as follows :

CALCULATION FOR $\bar{X}_2$

Marks (X_2)	*No. of Students* (f)	fX_2
5	4	20
10	6	60
15	8	120
20	7	140
25	3	75
30	2	60
	$N = \sum f = 30$	$\sum fX_2 = 475$

$$\therefore \quad \bar{X}_2 = \frac{\sum fX_2}{N} = \frac{475}{30} = 15.83$$

If the average marks of all students be denoted by $\bar{X}$, then we are given $\bar{X} = 68.6$. Thus, in terms of usual notations, we are given the following information :

	Passed	*Failed*	*Total*
Number of Students	$n_1 = 70$	$n_2 = 30$	$n_1 + n_2 = 100$
Mean Marks	$\bar{X}_1 = ?$	$\bar{X}_2 = 15.83$	$\bar{X} = 68.3$

Applying the formula $\bar{X} = \dfrac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$ for the combined mean, we obtain

$$68.6 = \frac{70\bar{X}_1 + 30 \times 15.83}{100} \Rightarrow 68.60 = 70\,\bar{X}_1 + 475$$

$$\Rightarrow \quad 70\,\bar{X}_1 = 6385 \quad \text{or} \quad \bar{X}_1 = 91.2$$

$\therefore$ Average marks of students who passed = 91.2.

EXAMPLE 88. *Find the geometric mean of 2, 4, 8, 12, 16 and 24.* **[*Delhi Univ. B.Com. 1985*]**

SOLUTION.

CALCULATION OF G.M.

X	$\log X$
2	0.3010
4	0.6021
8	0.9031
12	1.0792
16	1.2041
24	1.3802
$n = 6$	$\sum \log X = 5.4697$

$$G.M. = AL\left[\frac{1}{n}\sum \log X\right] = AL\left(\frac{5.4697}{6}\right) = AL\,(0.9116) = 8.159.$$

Calculation of Harmonic Mean - Individual Observations

The harmonic mean of a set of n observations $X_1, X_2, \ldots, X_n$ (not necessarily all disti
is given by

$$HM = \frac{n}{\frac{1}{X_1}+\frac{1}{X_2}\ldots+\frac{1}{X_n}} = \frac{n}{\Sigma\frac{1}{X}}$$

EXAMPLE 105. *Find the harmonic mean of 5 numbers 4, 5, 6, 10 and 12.*

SOLUTION. By definition,

$$HM = \frac{n}{\Sigma\frac{1}{X}} = \frac{5}{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{10}+\frac{1}{12}} = \frac{5}{\frac{15+12+10+6+5}{60}} = \frac{5\times 60}{48} = \frac{25}{4} = 6.25.$$

Calculation of Median - Individual Observations

For ungrouped data consisting of n observations, the calculation of median involves the following steps:

Step 1. *Arrange the given set of observations in an ascending or descending order of magnitude.*

Step 2. *The median is given by*

(i) *the value of* $\left(\frac{n+1}{2}\right)$ *th observation, when n is odd*

(ii) *the arithmetic mean of the values of* $\left(\frac{n}{2}\right)$ *th and* $\left(\frac{n}{2}+1\right)$ *th observations, when n is even.*

$$\frac{\frac{n}{2}+\left(\frac{n}{2}+1\right)}{2} = \frac{n+n+2}{2} = \frac{2n+2}{2} = n+1$$

EXAMPLE 40. *Find the median for the following data:*

(i)	*18*	*12*	*17*	*22*	*20*	*84*
(ii)	*85*	*69*	*74*	*60*	*59*	

SOLUTION. (*i*) Arranging the data in ascending order of magnitude, we get

12 17 18 20 22

Here, n = the number of observations = 5, an odd number

∴ median = size of $\left(\frac{n+1}{2}\right)$th observation = size of 3rd observation = 18

(*ii*) Arranging the data in ascending order of magnitude, we get

59 60 69 74 84 85

Here, n = the number of observations = 6, an even number

∴ median = Arithmetic mean of two middle terms

= Arithmetic mean of 3rd and 4th terms = $\frac{1}{2}(69+74)$ = 71.5

Calculation of Median — Discrete Series

In the case of discrete series, where the variable takes the values $X_1, X_2, \ldots, X_n$ wi[th] respective frequencies $f_1, f_2, \ldots, f_n$ with $\sum f = N$, median is the size of $\left(\frac{N+1}{2}\right)$[th] observation. In this case, the calculation of median involves the following steps:

Step 1. *Prepare the 'less than' cumulative frequency (c.f.) distribution.*

Step 2. *Find* $\frac{N+1}{2}$

Step 3. *See the c.f. just greater than or equal to* $\frac{N+1}{2}$.

Step 4. *The value of the variable corresponding to the c.f. obtained in Step 3 gives t[he] required median.*

EXAMPLE 41. *Calculate median from the following data:*

X :	*10*	*20*	*30*	*40*	*50*	*60*	*70*
f :	*1*	*5*	*12*	*20*	*19*	*9*	*4*

SOLUTION. **CALCULATION OF MEDIAN**

X	*f*	*Less than c.f.*
10	1	1
20	5	6
30	12	18
40	20	38
50	19	57
60	9	66
70	4	70

$N = \sum f = 70$

We have $\frac{N+1}{2} = \frac{71}{2} = 35.5$ and the c.f. just greater than or equal to 35.5 is 38. The corresponding value of X is 40.

$\therefore$ Median $= 40.$

Calculation of Median — Continuous Series

In the case of continuous series, median is the size of $\frac{N}{2}$ th observation, where $N = \sum f$ is the total frequency. The calculation of median in this case involves the following steps:

Step 1. *Prepare the 'less than' cumulative frequency (c.f.) distribution.*

Step 2. *Find* $\frac{N}{2}$.

Step 3. *See the c.f. just greater than or equal to* $\frac{N}{2}$.

Step 4. *Find the class corresponding to the c.f. obtained in Step 3. This is called the median class.*

Step 5. *Apply the following interpolation formula for calculating the median:*

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h,$$

where l = *lower limit of the median class,*

f = *frequency of the median class,*

C = *cumulative frequency of the class preceding the median class, and*

h = *size or width of the median class.*

NOTE. It may be noted that the interpolation formula used to obtain median is based on the following assumptions:

1. *The distribution of the variable under consideration is continuous with exclusive type classes without any gap.*
2. *There is an orderly and even distribution of observations within each class.*

EXAMPLE 42. *The marks obtained by 100 students in a certain examination are given below:*

Marks :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of Students :	10	9	25	30	16	10

Calculate the median marks.

SOLUTION.

CALCULATION OF MEDIAN

Marks	*No. of Students (f)*	*c.f. (less than)*
0 – 10	10	10
10 – 20	9	19
20 – 30	25	44
30 – 40	30	74 ← *Median class*
40 – 50	16	90
50 – 60	10	100
	$N = \sum f = 100$	

We have $\frac{N}{2} = \frac{100}{2} = 50$. The cumulative frequency just greater than or equal to 50 and the corresponding class interval is 30 – 40. Thus the median class is 30 – 40. median is given by the formula

$$Md = l + \frac{\frac{N}{2} - C}{f} \times h$$

where l = *lower limit of the median class* = 30

f = *frequency of the median class* = 30

C = *cumulative frequency of the class preceding the median class* = 44

h = *size of the median class* = 10

$$\therefore \quad Md = 30 + \frac{50 - 44}{30} \times 10 = 30 + 2 = 32 \text{ marks.}$$

EXAMPLE 43. *Given below is the distribution of marks obtained by 140 students in an examination*

Marks :	*10 – 19*	*20 – 29*	*30 – 39*	*40 – 49*	*50 – 59*	*60 – 69*	*70 – 79*	*80 – 89*	*9…*
No. of Students :	*7*	*15*	*18*	*25*	*30*	*20*	*16*	*7*	

Find the madian of the distribution. **[C.A. PEE-I, May…**

SOLUTION. Since data is given as a grouped frequency distribution with inclusive type classes, the first step involved in computation of median is to convert the given data into continuous frequency distribution with exclusive type classes as shown below:

CALCULATION OF MEDIAN

Class Boundaries	*Frequency (f)*	*c.f. (less than)*
9.5 – 19.5	7	7
19.5 – 29.5	15	22
29.5 – 39.5	18	40
39.5 – 49.5	25	65
49.5 – 59.5	30	95
59.5 – 69.5	20	115
69.5 – 79.5	16	131
79.5 – 89.5	7	138
89.5 – 99.5	2	140
	$N = \sum f = 140$	

We have $\frac{N}{2} = \frac{140}{2} = 70$. The cumulative frequency just greater than or equal to 70 and the corresponding class interval is 49.5 – 59.5. Thus the median class is 49.5 – … The median is given by the formula

$$Md = l + \frac{\frac{N}{2} - C}{f} \times h,$$

where $l = 49.5, \quad C = 65, \quad f = 30 \quad$ and $\quad h = 10$

∴ $$Md = 49.5 + \frac{70-65}{30} \times 10 = 49.5 + 1.67 = 51.17.$$

EXAMPLE 44. *Calculate median from the following data :*

Age	No. of persons	Age	No. of persons
55 – 60	7	35 – 40	30
50 – 55	13	30 – 35	33
45 – 50	15	25 – 30	58
40 – 45	20	20 – 25	14

SOLUTION. We first arrange the series in ascending order as shown in the following table :

CALCULATION OF MEDIAN

Age	No. of persons (f)	c.f. less than
20 – 25	14	14
25 – 30	28	42
30 – 35	33	75
35 – 40	30	105
40 – 45	20	125
45 – 50	15	140
50 – 55	13	153
55 – 60	7	160
	$N = \sum f = 160$	

Since $\frac{N}{2} = \frac{160}{2} = 80$ and c.f. just greater than or equal to 80 is 105, therefore median lies in the class 35 – 40 and is given by

$$Md = l + \frac{\frac{N}{2} - C}{f} \times h,$$

where $l = 35, \quad C = 75, \quad f = 30 \quad \text{and} \quad h = 5$

∴ $$Md = 35 + \frac{80-75}{30} \times 5 = 35 + 0.83 = 35.83$$

REMARK. ***Calculation of Median when Class Intervals are Unequal***

It may be remarked that even if class intervals are unequal in size, the frequencies need not be adjusted to make the class intervals equal and the same interpolation formula can be applied for calculating median as discussed before.

EXAMPLE 45. *Calculate median from the following data:*

Marks :	*0 – 10*	*10 – 30*	*30 – 50*	*50 – 60*	*60 – 80*	*80 – 90*
No. of Students :	*5*	*15*	*20*	*10*	*8*	*2*

SOLUTION.

CALCULATION OF MEDIAN

Marks	*No. of Students (f)*	*c.f. (less than)*
0 – 10	5	5
10 – 30	15	20
30 – 50	20	40
50 – 60	10	50
60 – 80	8	58
80 – 90	2	60
	$N = \sum f = 60$	

Since $\frac{N}{2} = 30$ and c.f. just greater than or equal to 30 is 40, therefore median class 30 – 50. Using the following formula for median,

$$Md = l + \frac{\frac{N}{2} - C}{f} \times h,$$

where $l = 30$, $C = 20$, $f = 20$ and $h = 20$, we get

$$Md = 30 + \frac{30 - 20}{20} \times 10 = 40.$$

EXAMPLE 47. *An incomplete distribution is given below:*

Class :	*0 – 10*	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*	*60 – 70*	*Total*
Frequency :	*10*	*20*	*?*	*40*	*?*	*25*	*15*	*170*

Find out missing frequencies if median value is 35. **[Kerala Univ. B.Com. 2004]**

SOLUTION. Let the missing frequencies for the classes 20 – 30 and 40 – 50 be f_1 and f_2 respectively. To find f_1 and f_2, we prepare the following table.

Class Interval	*f*	*c.f.*
0 – 10	10	10
10 – 20	20	30
20 – 30	f_1	$30 + f_1$
30 – 40	40	$70 + f_1$
40 – 50	f_2	$70 + f_1 + f_2$
50 – 60	25	$95 + f_1 + f_2$
60 – 70	15	$110 + f_1 + f_2$
	$N = \sum f = 110 + f_1 + f_2$	

We are given $N = \sum f = 170$

$$\Rightarrow \quad 110 + f_1 + f_2 = 170 \quad \Rightarrow \quad f_1 + f_2 = 60 \quad \text{...(1)}$$

Since median is given to be 35, which lies in the class 30 – 40, therefore 30 – 40 is the median class. Applying the formula for computing median, we get

$$35 = 30 + \frac{85 - (30 + f_1)}{40} \times 10 \quad \Rightarrow \quad 5 = \frac{55 - f_1}{4}$$

$$\Rightarrow \quad 55 - f_1 = 20 \quad \text{or} \quad f_1 = 35$$

Substituting $f_1 = 35$ in (1), we get $f_2 = 25$. Hence missing frequencies are 35 and 25 respectively.

Calculation of Median in Cumulative Series

If the data is given in the form of a cumulative frequency distribution, it has to be first arranged in an ordinary frequency distribution in order to find out the frequency of the median class which is needed in the calculation of median. Once it is done, the rest of the procedure is same as in any other continuous series.

EXAMPLE 51. *Following is the distribution of marks obtained by 125 students in a Business Statistics paper :*

Marks (less than) :	*10*	*20*	*30*	*40*	*50*	*60*	*70*	*80*
No. of Students :	*4*	*16*	*40*	*76*	*96*	*112*	*120*	*125*

Calculate the median marks.

SOLUTION. Since the data is given in the form of cumulative frequency distribution, it has to be arranged in a frequency distribution as shown in the following table :

CALCULATION OF MEDIAN

Marks	*Frequency*	*c.f.*
0 – 10	4	4
10 – 20	16 – 4 = 12	16
20 – 30	40 – 16 = 24	40
30 – 40	76 – 40 = 36	76
40 – 50	96 – 76 = 20	96
50 – 60	112 – 96 = 16	112
60 – 70	120 – 112 = 8	120
70 – 80	125 – 120 = 5	125 = N

Since $\frac{N}{2} = \frac{125}{2} = 62.5$ and *c.f.* just greater than or equal to 62.5 is 76, therefore median lies in the class 30 – 40 and is given by

$$Md = l + \frac{\frac{N}{2} - C}{f} \times h,$$

where $l = 30$, $C = 40$, $f = 36$ and $h = 10$

$$\therefore \quad Md = 30 + \frac{62.5 - 40}{36} \times 10 = 30 + 62.5 = 36.25.$$

EXAMPLE 52. *Following is the distribution of marks obtained by 65 students in statistics paper:*

Marks (more than):	*20*	*30*	*40*	*50*	*60*	*70*
No. of Students:	*65*	*63*	*40*	*40*	*18*	*7*

Calculate the median marks.

SOLUTION. Since the data is given in the form of cumulative frequency distribution, it has to be arranged in a frequency distribution as shown in the following table :

CALCULATION OF MEDIAN

Marks	*Frequency (f)*	*c.f. (less than)*
20 – 30	65 – 63 = 2	2
30 – 40	63 – 40 = 23	25
40 – 50	40 – 40 = 0	25
50 – 60	40 – 18 = 22	47 ← *Median class*
60 – 70	18 – 7 = 11	58
70 and above	7	65 = N

$$\text{Median} = \text{size of } \frac{N}{2}\textit{th} \text{ item} = \text{size of } 32.5 \text{ item}$$

∴ Median lies in the class 50 – 60 and is given by

$$Md = l + \frac{\frac{N}{2} - C}{f} \times h = 50 + \frac{32.5 - 25}{22} \times 10 = 50 + 3.41 = 53.41.$$

Calculation of Mode - Ungrouped Data

For determining mode in the case of ungrouped data, count the number of times the various values repeat themselves and the value occurring the maximum number of times is the mode.

For example, the mode of the set of numbers

3 4 4 5 6 7 7 7 9

is 7, since it appears three times and no other value appears more than twice.

Calculation of Mode - Discrete Series

In discrete frequency distribution, mode can be determined just by inspection. It is the value of the variable corresponding to the maximum frequency. However, this method is applicable only if the distribution is 'unimodal', *i.e.*, if it has only one mode. For example, consider the following distribution:

X	:	1	2	3	4	5	6	7
f	:	1	4	12	7	2	3	1

Since the value of X corresponding to the maximum frequency is 3, the mode is 3.

NOTE. While determining mode by inspection in the case of discrete frequency distribution, an error of judgment is possible when the difference between the greatest frequency and the frequency preceding it or succeeding it is very small and the values are heavily concentrated on either side. In such cases, it is desirable to locate the mode by what is called the *method of grouping.*

Method of Grouping

The method of grouping involves preparing a grouping table. A grouping table has six columns. In column (1), we write down the original frequencies. The greatest frequency in this column is put in a circle or marked by bold type. In column (2), frequencies are grouped in two's. In column (3), we leave the first frequency and then group the remaining frequencies in two's. In column (4), frequencies are grouped in three's. In column (5), we leave the first frequency and then group the remaining frequencies in three's. In column (6), we leave the first two frequencies and then group the remaining frequencies in three's. In each of these columns, the highest total is put in a circle or marked by bold type.

After completing the grouping table, we prepare an analysis table. In the analysis table, column numbers are put on the left-hand side and the various probable values of mode are put on the right-hand side. The values against which frequencies are highest are entered by means of a bar in the relevant box corresponding to the values they represent. The value which is repeated the maximum number of times represents the mode.

The method of preparing grouping table and analysis table is best illustrated in the following example.

EXAMPLE 70. *Calculate mode from the following data:*

Height in inches :	56	58	59	60	61	62	63	64	66	68
No. of Persons :	3	7	6	9	20	22	24	5	3	1

SOLUTION. By inspection one is likely to say that the mode is 63 since it occurs the maxim number of times, *i.e.*, 24. However, the difference between the maximum frequency a the frequency preceding it is very small, we prepare a grouping table and an analy table as shown below :

GROUPING TABLE

Height in inches	*Col. 1*	*Col. 2*	*Col. 3*	*Col. 4*	*Col. 5*	*Col. 6*
56	3	10		16		
58	7		13		22	
59	6	15				35
60	9		29	51		
61	20	42			66	
62	22		46			51
63	24	29		32		
64	5		8		9	
66	3	4				
68	1					

ANALYSIS TABLE

Col. No.	56	58	59	60	61	62	63	64	66	68
1							1			
2					1	1				
3						1	1			
4				1	1	1				
5					1	1	1			
6						1	1	1		
Total				1	3	5	4	1		

Since the value 62 has occurred the maximum number of times, *i.e.*, 5, the mode

Calculation of Mode - Continuous Frequency Distribution

In the case of continuous frequency distribution, the first step is to ascertain the modal class, *i.e.*, the class corresponding to the maximum frequency. This can be done either by inspection or by preparing the grouping table and analysis table. The value of mode is then obtained by applying the following interpolation formula:

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h,$$

where l = lower limit of the modal class,

f_1 = frequency of the modal class,

f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class,

h = size of the modal class. (or class - interval)

The above formula for calculating mode can also be put in a different form as follows:

$$\text{Mode} = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h,$$

where $\Delta_1 = f_1 - f_0$ = frequency of the modal class – frequency of the class preceding the model class

$\Delta_2 = f_1 - f_2$ = frequency of the modal class – frequency of the class succeeding the modal class

REMARK. It may be remarked that the above formula for computing mode is based on the following assumptions:

1. *The frequency distribution must be continuous with exclusive type classes without any gaps. If the data are not given in the form of continuous classes, it must first be converted into continuous classes before applying the above formula.*
2. *The class intervals must be uniform throughout, i.e., the size of all the class intervals must be same. If they are unequal they should first be made equal on the assumption that frequencies are uniformly distributed over all the classes.*

EXAMPLE 71. *Compute the mode for the following distribution:*

Class Interval	:	*0 – 8*	*8 – 16*	*16 – 24*	*24 – 32*	*32 – 40*	*40 – 48*
Frequency	:	*8*	*7*	*16*	*24*	*15*	*7*

SOLUTION. The class corresponding to the maximum frequency, 24, is 24 – 32. Thus the modal class is 24 – 32. Applying the interpolation formula for computing mode:

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h,$$

where l = lower limit of the modal class = 24

f_1 = frequency of the modal class = 24

f_0 = frequency of the class preceding the modal class = 16

f_2 = frequency of the class succeeding the modal class

h = size of the modal class = 8

$$\therefore \quad \text{Mode} = 24 + \frac{24-16}{48-16-15} \times 8 = 24 + \frac{8}{17} \times 8 = 24 + 3.76 = 27.76$$

EXAMPLE 72. *Given below is the distribution of weights of a group of 60 students in a*

Weights (in kg) :	*30 – 34*	*35 – 39*	*40 – 44*	*45 – 49*	*50 – 54*	*55 – 59*
No. of Students :	3	5	12	18	14	6

Find the mode of the distribution.

SOLUTION. Since the formula for mode requires the distribution to be continuous 'exclusive type' classes, we first convert the classes into class boundaries as shown in following table:

CALCULATION OF MODE

Weight (in kg)	*Class Boundary*	*No. of Students (f)*
30 – 34	29.5 – 34.5	3
35 – 39	34.5 – 39.5	5
40 – 44	39.5 – 44.5	12
45 – 49	44.5 – 49.5	18
50 – 54	49.5 – 54.5	14
55 – 59	54.5 – 59.5	6
60 – 64	59.5 – 64.5	2

Since the maximum frequency is 18, therefore the corresponding class 44.5 – 49.5 is modal class. Applying the modal formula:

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h,$$

where, $l = 44.5$, $f_1 = 18$, $f_0 = 12$, $f_2 = 14$ and $h = 5$, we get

$$\text{Mode} = 44.5 + \frac{18-12}{36-12-14} \times 5 = 44.5 + \frac{6}{10} \times 5 = 44.5 + 3 =$$

Mode when Class intervals are unequal. The formula for calculating the value of mode given above is applicable only where there are equal class intervals. If the class intervals are unequal, then we must make them equal before we start computing the value of mode. The class interval should be made equal and frequencies adjusted on the assumption that they are equally distributed throughout the class.

EXAMPLE 73. *Calculate mode from the following data :*

Marks :	*0 – 10*	*10 – 20*	*20 – 40*	*40 – 50*	*50 –*
No. of Students :	2	7	18	15	

SOLUTION. Since the class intervals are unequal, they should first be made equal by adjusting the frequencies.

CALCULATION OF MODE

Marks	*No. of students (f)*
0 – 10	2
10 – 20	7
20 – 30	9
30 – 40	9
40 – 50	15
50 – 60	4
60 – 70	4

By inspection the class 40 – 50 is the modal class. Applying the mode formula:

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h,$$

where, $l = 40$, $f_1 = 15$, $f_0 = 9$, $f_2 = 4$ and $h = 10$, we get

$$\text{Mode} = 40 + \frac{15-9}{30-9-4} \times 10 = 40 + \frac{6}{17} \times 10 = 40 + \frac{60}{17} = 40 + 3.53 = 43.53$$

NOTE. If we had not made any adjustment, the value of mode would have been

$$\text{Mode} = 20 + \frac{18-7}{36-7-15} \times 20 = 20 + \frac{11}{14} \times 20 = 20 + \frac{110}{14} = 20 + 15.71 = 35.71,$$

which is not possible, since mode cannot be less than 40.

EXAMPLE 76. *The distribution of age of patients turned out in a hospital on January 1, 2005 was as under :*

Age (in years)	*No. of Patients*
more than 10	*148*
more than 20	*124*
more than 30	*109*
more than 40	*71*
more than 50	*30*
more than 60	*16*
more than 70 and upto 80	*01*

Find the median age and modal age of the patients. **[*C.A. PEE-I, May 2005*]**

SOLUTION. Since the data is given in the form of a cumulative frequency distribution, it has to be first arranged in a frequency distribution as shown in the following table :

CALCULATION OF MEAN AND MODE

Age (in years)	*No. of patients (f)*	*c.f. (less than)*	
10 – 20	148 – 124 = 24	24	
20 – 30	124 – 109 = 15	39	
30 – 40	109 – 71 = 38	77	← Median class
40 – 50	71 – 30 = 41	118	← Modal class
50 – 60	30 – 14 = 14	132	
60 – 70	16 – 1 = 15	147	
70 – 80	01	148	= *N*

Calculation of Median :

$$\text{Median} = \text{size of } \frac{N}{2} \text{th item} = \text{size of } \frac{148}{2} = 74\text{th item.}$$

∴ Median lies in the class 30 – 40.

Applying the following formula for median :

$$Md = l + \frac{\frac{N}{2} - C}{f} \times h,$$

where $l = 30$, $C = 39$, $f = 38$, and $h = 10$, we get

$$Md = 30 + \frac{74 - 39}{38} \times 10 = 30 + 9.21 = 39.21$$